Deriving the Beta Function for the Cubing Operator in the Trikaya Wheel

In renormalization group (RG) theory, the beta function β(g) describes how a coupling g "runs" (flows) with the energy scale μ:

$$\beta(g) = \mu \frac{dg}{d\mu}$$

Positive β → relevant (grows toward IR), negative → irrelevant (damps to zero), zero → marginal/fixed.

Our Trikaya RG operator \mathcal{R}(v) = v^3 is a discrete coarse-graining step: cubing damps amplitudes while tripling frequencies. To derive an analogous **beta function** for cubing, we treat the amplitude A of the resonant signal as the "coupling" and model the discrete flow under iteration.

1. Discrete Flow Under Cubing

Consider a dominant mode v(t) ≈ A sin(ω t).

Apply \mathcal{R}:

$$R(v) = A^3 \sin^3(\omega t) = A^3 \frac{3\sin(\omega t) - \sin(3\omega t)}{4}$$

The new amplitude of the fundamental (ω) mode is (3/4) A^3, while a tripled-frequency mode appears.

After one step, effective amplitude scaling:

$$A' = \frac{3}{4} A^3$$

(This ignores the new 3ω mode for the fundamental beta; higher modes have their own flows.)

2. From Discrete to Continuous Beta Function

To obtain a continuous analog, think of each cubing as changing the scale by a factor b = 3 (tripling "lattice spacing" or frequency).

The discrete change:

$$A(b\mu) = \frac{3}{4}A(\mu)^3$$

Take logarithm for small changes. Define the flow with discrete log-scale step ln b = ln 3:

$$A_{n+1} = \frac{3}{4}A_n^3$$

For small deviations around a reference, approximate the flow as differential.

Let t = ln μ (RG "time," increasing toward IR), dt = ln 3 per step.

Then:

$$\frac{A(t+\Delta t) - A(t)}{\Delta t} \approx \frac{\frac{3}{4}A^3 - A}{\ln 3}$$

In the continuum limit:

$$\frac{dA}{dt} = \frac{1}{\ln 3}\left(\frac{3}{4}A^3 - A\right)$$

Thus, the Trikaya beta function for the amplitude coupling A is:

$$\beta(A) = \frac{1}{\ln 3}\left(\frac{3}{4}A^3 - A\right) = \frac{A}{\ln 3}\left(\frac{3}{4}A^2 - 1\right)$$

3. Fixed Points and Stability

Set β(A) = 0:

$$A = 0 \quad \text{or} \quad A^2 = \frac{4}{3} \Rightarrow A = \pm\sqrt{\frac{4}{3}} \approx \pm 1.154$$

A* = 0: Attractive for |A| < √(4/3) (irrelevant directions damp to nullity).

A* = ±√(4/3): Attractive for larger |A| (relevant growth stabilized).

This matches our earlier discrete fixed points (adjusted by the 3/4 normalization).

4. Interpretation

- Negative β for small A → flow to nullity (IR silence).
- Positive β for large A → growth until balanced by higher terms (in full system, capped by nullity bounds).
- The 1/ln3 factor reflects the discrete step size (tripling scale).

This beta function formalizes the Trikaya flow: cubing drives irrelevant amplitudes to the nullity fixed point while preserving resonant maxima—exactly the ladder's damping to silence.

The wheel's beta sings toward nullity.